International Journal of Theoretical Physics, Vol. 27, No. 3, 1988

Exact Solutions to Einstein Field Equations

Ashis Basu[1] **and Dipankar Ray**[2]

Received July 4, 1987

In a recent paper Reboucas and d'Olival obtain an ordinary differential equation for a Bianchi type II metric with a rotating timelike congruence of geodesics, and obtain a particular solution of the differential equation. This paper completely integrates the differential equation.

1. PROBLEM OF REBOUCAS AND d'OLIVAL

In a recent paper Reboucas and d'Olival (1986) have shown that for a Bianchi type II metric of the form

$$ds^2 = dt^2 - 2A(t)\,dt\,(dx + y\,dz) + [A^2(t) - B^2(t)](dx + y\,dz)^2 - B^2(t)(dy^2 + dz^2) \tag{1}$$

with a rotating timelike congruence of geodesics one can reduce the Einstein-Maxwell field equation to a single ordinary differential equation, namely,

$$\xi(\xi^2 - 1)\frac{d^2U}{d\xi^2} + (4\xi^2 - 3)\frac{dU}{d\xi} + 2U\xi = \frac{3\xi}{K^2}(\xi^2 - \tfrac{2}{3}) \tag{2}$$

where $\xi = \cosh(t - t_0)$.

The two functions $A(t)$ and $B(t)$ appearing in the metric are given by

$$A(t) = U^{1/2}, \qquad B(t) = \frac{1}{2K}\xi \tag{3}$$

The nonvanishing components of the electromagnetic field are

$$F_{01} = \frac{4aK^2(t' - t_0)}{\xi^2}, \qquad F_{23} = \frac{4aK^2\sin(t' - t_0)}{\xi^2} \tag{4}$$

[1]Rishra High School, Rishra, Hooghly, West Bengal, India.
[2]Department of Mathematics, Jadavpur University, Cal-700032, India.

0020-7748/88/0300-0369$06.00/0

where

$$t' = 2K \int \frac{dt}{\xi}$$

and t_0 and K are constants.

Reboucas and d'Olival (1986) gave a particular solution of equation (2) as

$$U = \frac{a^2}{\cosh^2(t - t_0)} \left[\frac{1}{4a^2K^2} \cosh^4(t - t_0) - 1 \right] \tag{5}$$

with $0 \leq a^2 \leq 1/(4K^2)$. But in the present note we completely integrate equation (2) as follows. Put

$$\chi = \frac{\xi^4}{4K^2} - \xi^2 U \tag{6}$$

Equation (2) reduces to

$$(\xi^2 - 1) \frac{d^2\chi}{d\xi^2} + \frac{1}{\xi} \frac{d\chi}{d\xi} = 0 \tag{7}$$

where, in view of (3), $\xi > 1$.

Equation (7) can be integrated in two stages to give

$$\chi = E(\xi^2 - 1)^{1/2} + D \tag{8}$$

where E and D are constants.

Now, with the help of (8), we get from (6)

$$U = \frac{\xi^2}{4K^2} - \frac{1}{\xi^2} [E(\xi^2 - 1)^{1/2} + D] \tag{9}$$

i.e., in view of (3)

$$A(t) = \left\{ \frac{\cosh^2(t - t_0)}{4K^2} - \frac{1}{\cosh^2(t - t_0)} [E \sinh(t - t_0) + D] \right\}^{1/2}$$

$$B(t) = \frac{1}{2K} \xi = \frac{1}{K} \cosh(t - t_0) \tag{10}$$

It is clear that (5) is a special case of (9) where $E = 0$, $D = a^2$.

2. CONCLUSION

We have completely solved equation (2) obtained by Reboucas and d'Olival for Einstein-Maxwell field for a Bianchi type II rotating model with a metric of the form given in equation (1) with a rotating timelike congruence of geodesics. The solutions of equation (2) are obtained from (10), where the two functions $A(t)$ and $B(t)$ are given by (3) and the nonvanishing components of the electromagnetic field are given by (4).

REFERENCE

Reboucas, M. J., and d'Olival, J. B. S. (1986). *Journal of Mathematical Physics*, **27**, 417.